REVERSING A TRAILER – TENDENCY TO JACK-KNIFE

Have you ever wondered why it is much easier to reverse a caravan than a short camping trailer – even if you can see the trailer out of your rear window, and don't have to wait for it to appear in a wing mirror?! Or have you ever wondered why HGV drivers make it look so easy to reverse a 30' articulated semi-trailer?

As anyone who has successfully reversed a trailer round a corner will know, you have to begin by turning the steering wheel the "wrong" way, and when the trailer is pointing in roughly the right direction, you then steer the other way to "chase" it round – making small steering corrections as necessary. Although this undoubtedly takes skill, the degree of difficulty can vary significantly between one outfit and another. Why?

The answer lies in the fact that many (but not all) outfits have a "critical" angle. This is the angle between the towcar's and trailer's longitudinal axes beyond which you cannot straighten up. Beyond this angle, if you continue to reverse – even with the towcar on full lock – the trailer will jack-knife. The larger this critical angle, the easier it is to reverse the outfit – because if the trailer is turning too sharply, you have a much greater chance of correcting it.

The factors which increase this critical angle are the towcar's turning circle (the smaller the better), the trailer's longitudinal distance from axle to hitch (the longer the better) and the towcar's rear overhang – i.e. the longitudinal distance from rear axle to towball (the longer the better – although rear overhang is not always good for general towing stability!). If the trailer's axle to hitch dimension is greater than the towball's turning radius, there is no longer a critical angle, and recovery is possible from all angles – even beyond 90^{o}. [This is why long articulated vehicles are able to achieve some incredible manoeuvring feats with relative ease].

However, if you're trying to reverse a short camping trailer with a towcar which has a poor turning circle and its rear wheels very near the back, you're on a hiding to nothing! If you can't see the trailer out of your back window, forget it – because by the time it appears in your door mirror, it's too late!

I have long believed that it must be possible to calculate the critical angle for a given outfit, but have looked in vain for any information about this on the Internet. I have therefore been left with no alternative but to develop the necessary mathematics myself, and the results of this are set out below.

The geometry of the towcar is shown in Figure 1.

Assuming the steering geometry to conform with Ackerman principles, the normals from the two front wheels will intersect at a point on the line of the rear axle (Point O in Figure 1). All points on the towcar will rotate about this point. It is necessary to calculate the position of this point at full lock, and thence to determine the path of the towball.

In Figure 1, A is the centrepoint of the front outer wheel's contact patch and C is the corresponding point for the rear outer wheel (or, in the case of unequal front and rear tracks, is a point on the rear axle centreline, at the same distance as Point A from the vehicle's longitudinal centreline). B is the centre of the towball, and D is the centre point of the rear axle. [All projected onto a horizontal plain]

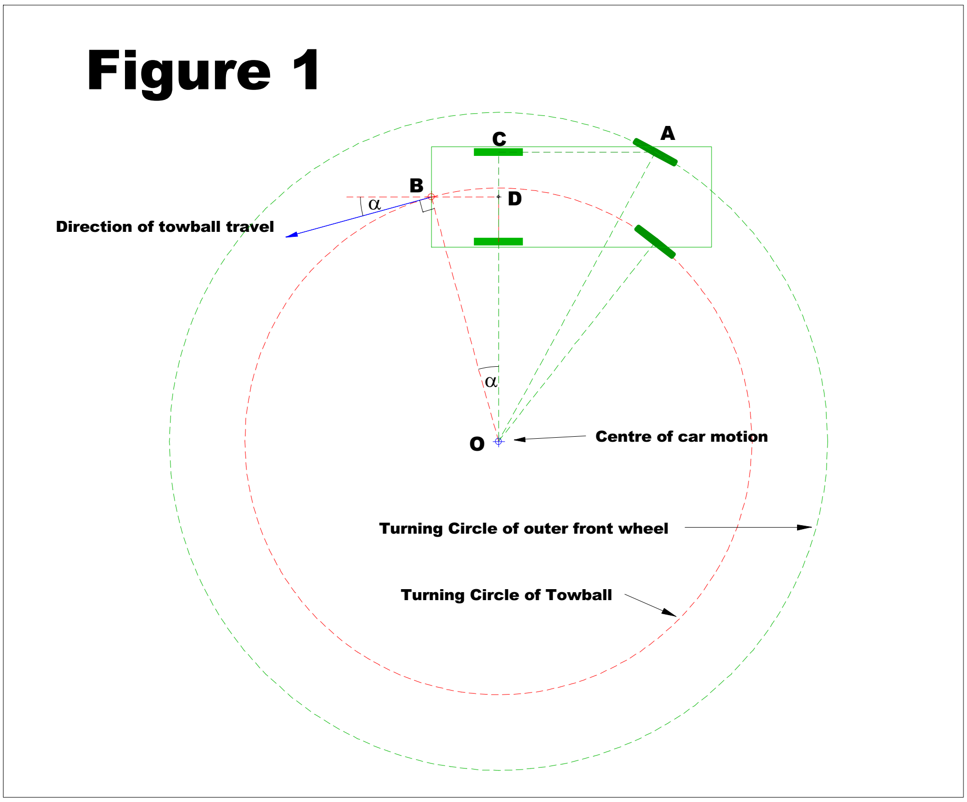
Let us define ***k*** as the kerb-to-kerb turning circle, ***d*** as the distance of the towball behind the rear axle (overhang), ***f*** as the tyre width, ***w*** as the wheelbase and ***t*** as the track (or, in the case of unequal front and rear tracks, as the Front track). [All of these should be available from a vehicle's specification].

The line OA represents the kerb-to-kerb turning radius all but half a tyre width – since A is at the centre of the tyre, while ***k*** is measured to the outside of the tyre. If the length of OA is r_a, then:

$$r_a = \frac{k - f}{2} \qquad \text{Eqn: 1}$$

If the length of OC is $\mathbf{r_c}$, this can then be calculated by Pythagoras – viz:

$$r_c = \sqrt{r_a^2 - w^2} \qquad \text{Eqn: 2}$$



If the distance OD is r_d, this can then be calculated by subtracting half the track, viz:

$$r_d = r_c - \frac{t}{2} \qquad \text{Eqn:3}$$

The radius of the path followed by the towball r_b can then be calculated by Pythagoras, using the overhang d (BD in Figure 1), viz:

$$r_b = \sqrt{r_d^2 + d^2} \qquad \text{Eqn: 4}$$

The direction of travel of the towball is tangential to the circle centred on point O, and thus makes an angle of α with the towcar's longitudinal axis, where:

$$\alpha = \tan^{-1}\frac{d}{r_d} \qquad \text{Eqn:5}$$

The geometry of the trailer is shown in Figure 2.

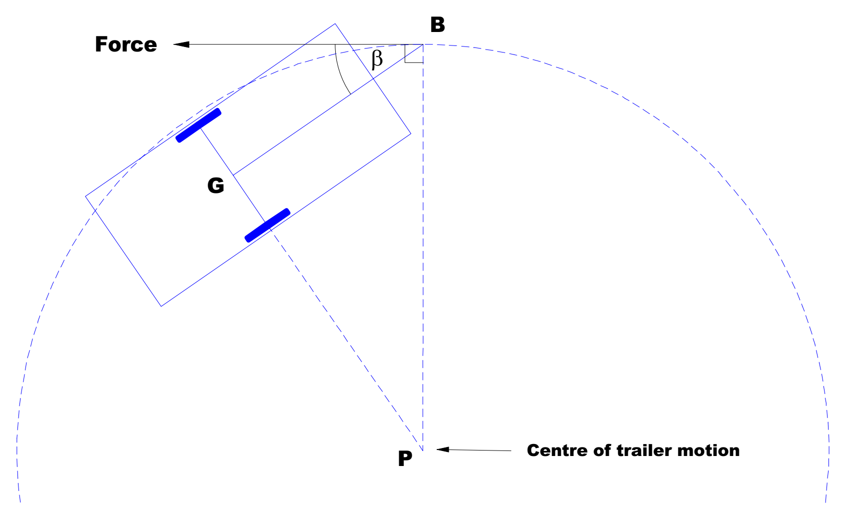
If a force is applied to the hitch (Point B) at an angle of β to the longitudinal axis of the trailer, the trailer will move around Point P – which is at the intersection of the normal to the force with the line of the axle.

The two extreme cases of this are:

(i) Force in line with longitudinal axis (i.e. $\beta = 0$)
In this case, the normal is parallel with the axle and only intersects it at infinity. The trailer thus moves in an arc of infinite radius – i.e. in a straight line backwards

(ii) Force normal to longitudinal axis (i.e. $\beta = 90^o$)
In this case, the normal to the force is in line with the longitudinal axis, and intersects the axle at its mid-point (Point G in Figure 2). The trailer thus rotates about Point G, and turns round within its own length.

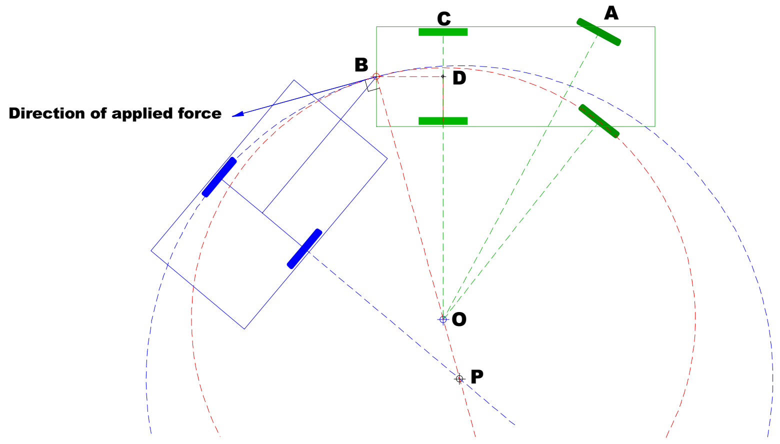
Figure 2



The combined effect of the geometries of the towcar and trailer are shown in Figures 3, 4 and 5. In all cases, the trailer is at an angle with the towcar, and the towcar is using full lock in an attempt to chase the trailer and straighten up. In all cases, the direction of the force applied to the trailer is tangential to the towball's turning circle, as defined by the towcar's geometry.

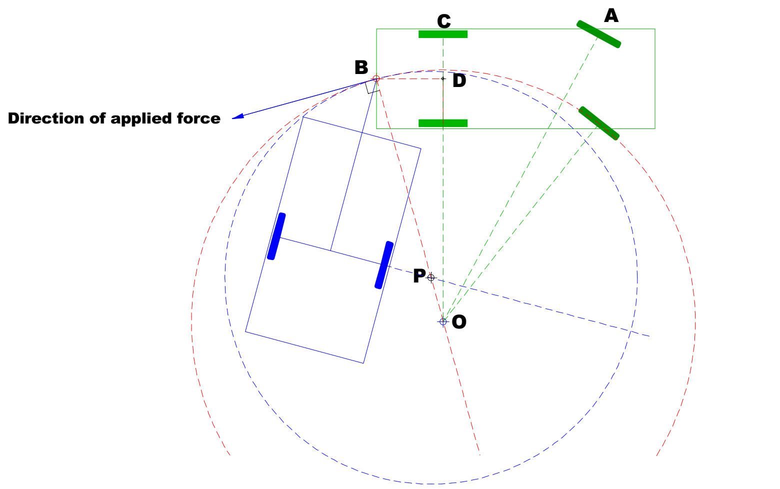
In Figure 3, the angle between the towcar and trailer is such that Point P is outboard of Point O – so that the trailer moves through a larger arc than the towcar. Thus the towcar has a higher angular velocity than the trailer in plan view, so as to reduce the angle between them. This situation is stable, and the towcar is able to straighten up.

Figure 3

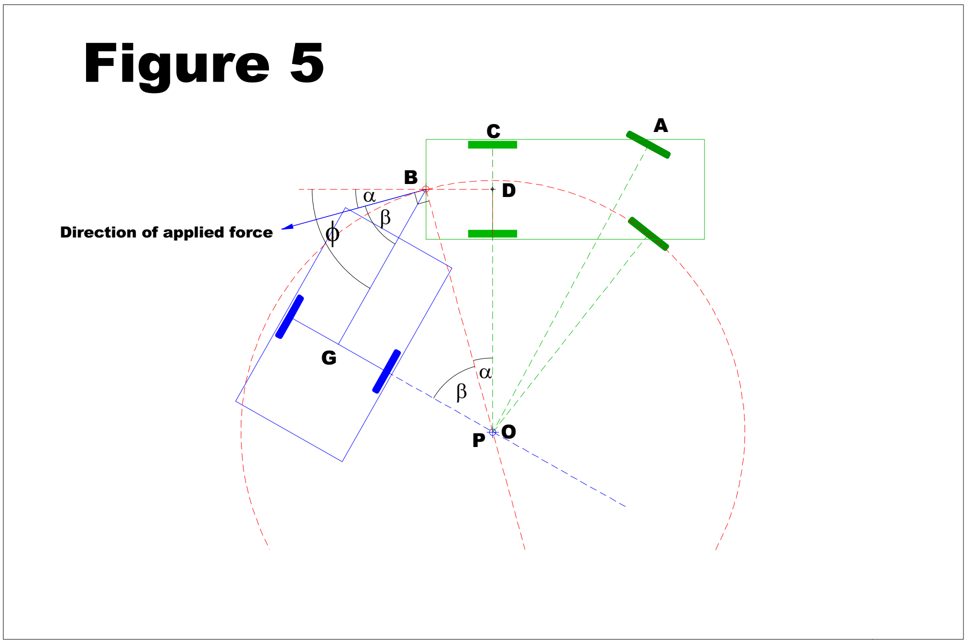


In Figure 4, the angle between the towcar and trailer is such that Point P is inboard of Point O – so that the trailer moves through a smaller arc than the towcar. Thus the towcar has a lower angular velocity than the trailer in plan view, so as to increase the angle between them. This situation is unstable, and results in jack-knifing.

Figure 4



In Figure 5, the angle between the towcar and trailer is such that Points P and O are co-incident – so that the trailer and towcar have equal angular velocities in plan view. The angle between them thus remains constant, and this angle is the *critical* angle.



As can be seen from Figure 5, the critical angle:

$$\phi = \alpha + \beta \qquad \text{Eqn: 6}$$

where α is as derived in Eqn 5, and where:

$$\beta = \sin^{-1} \frac{l}{r_b} \qquad \text{Eqn: 7}$$

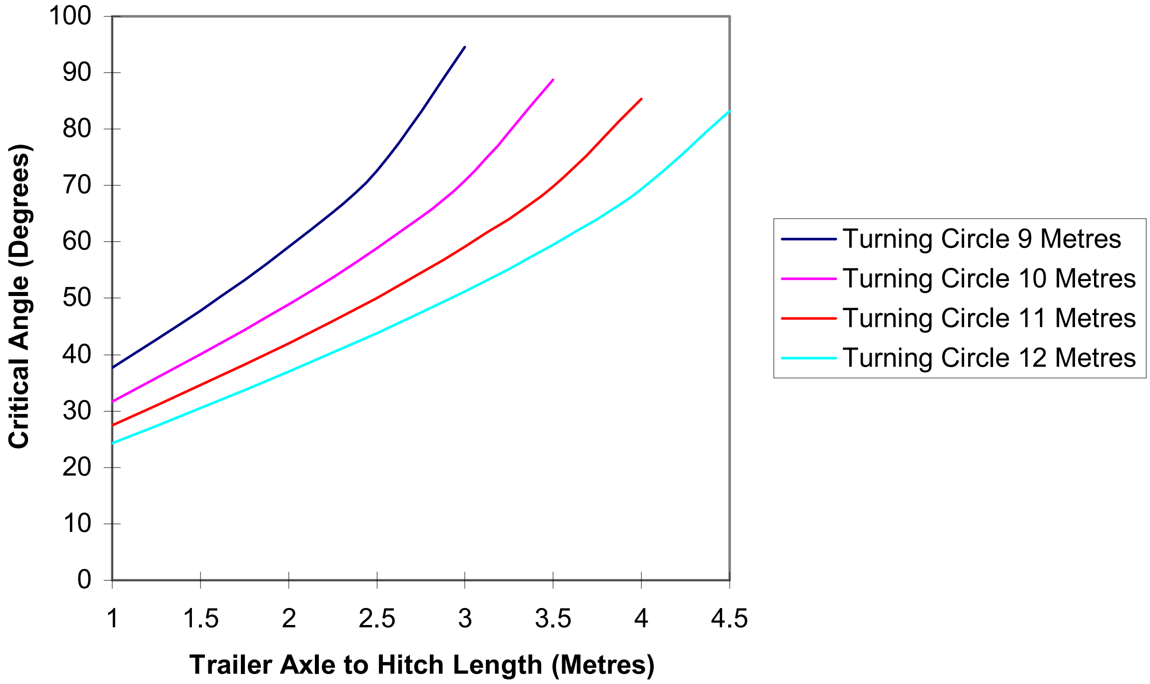
$\boldsymbol{l}$ is trailer axle to hitch distance (BG in Figure 5), and $\boldsymbol{r_b}$ is as derived in Eqn: 4.

It will be seen that β has no solution if $\boldsymbol{l} > \boldsymbol{r_b}$. This represents the case where the trailer hitch to axle distance is greater than the radius of travel of the towball, so that there is no critical angle.

Graph 1 shows the critical angles for towcars with a turning circle of 9, 10, 11 and 12 metres when pulling trailers whose axle to hitch distance ranges between 1 and 4.5 metres. In all cases, the towcar's wheelbase, track and tyre width has been taken to be 2.5, 1.4 and 0.185 metres respectively, and the rear axle to towball overhang has been taken as 1 metre.

It will be noted that with a large turning circle and short trailer, the critical angle can be less than 30^o – making manoeuvring very difficult. On the other hand, with a long trailer, there is either no critical angle or it is very large – making manoeuvring very much easier.

GRAPH 1 - Critical Angle vs Trailer Length



Even where there is no critical angle – or where there *is* a critical angle but the trailer is being operated well within it – the ease of manoeuvring in reverse is still to some extent determined by the geometry. This is because the distance which has to be travelled in order to achieve a given angular change depends on the separation between Points O and P. It will be remembered that when these points are co-incident, the outfit moves in a constant arc – neither jack-knifing nor straightening. If Point P is only just outboard of Point O, the rate of straightening will be very small – and the manoeuvrability very poor, because more than a slight departure from full lock will result in a jack-knife.

A reasonable measure of manoeuvrability would be amount of straightening per unit distance travelled.

In Figure 6, the angle between the towcar and trailer is θ. If $\boldsymbol{r_t}$ is the turning radius of the towball about Point P (PB in Figure 6), then:

$$r_t = l \operatorname{cosec} \beta \qquad \text{Eqn: 8}$$

where:

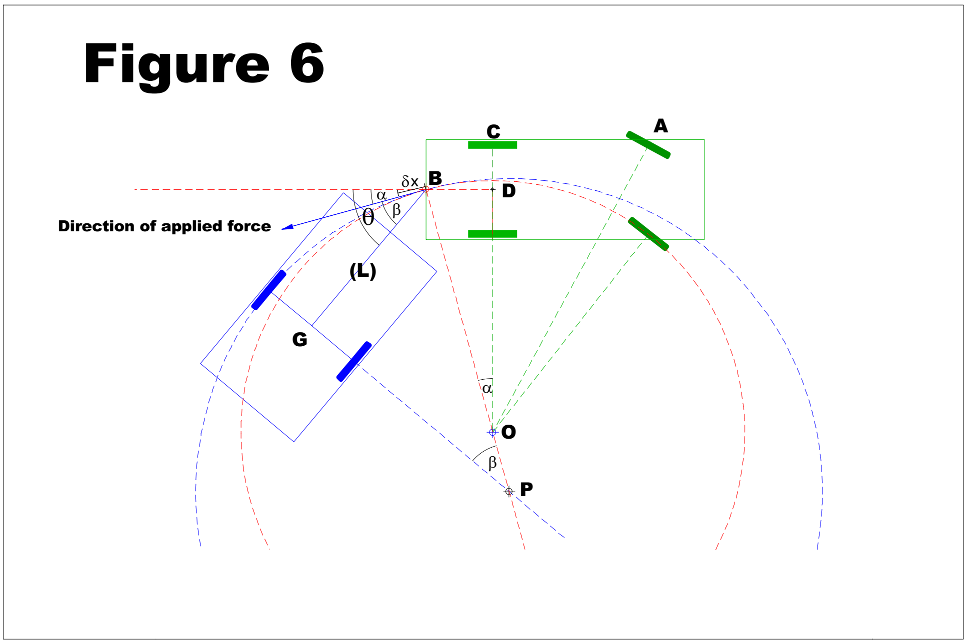
$$\beta = \theta - \alpha \qquad \text{Eqn: 9}$$

If the towball moves a distance $\delta \boldsymbol{x}$ along the tangent to the circle centred on Point O, the plan view angle of the towcar will change by $\delta \boldsymbol{x} / \boldsymbol{r_b}$ (radians). The plan view angle of the trailer will change by $\delta \boldsymbol{x} / \boldsymbol{r_t}$. The amount of straightening ($\delta\theta$) is thus:

$$\delta\theta = \frac{\delta x}{r_b} - \frac{\delta x}{r_t} = \delta x \frac{(r_t - r_b)}{r_t r_b} \qquad \text{Eqn: 10}$$

If divided by $\delta \boldsymbol{x}$, this yields the amount of straightening per unit distance travelled:

$$\frac{d\theta}{dx} = \frac{(r_t - r_b)}{r_t\, r_b} \qquad \text{Eqn: 11}$$



The parameters affecting non-jack-knifing manoeuvrability can most easily be illustrated by reference to some real-life examples. Graph 2 below shows the rate of straightening vs towcar/trailer alignment angle for various lengths of trailer attached to a Volvo V70 (old shape). The following base data have been used:

Wheelbase:	2.66 metres
Track:	1.47 metres
Turning Circle:	10.2 metres
Tyre width:	0.195 metres [195/65-15 tyres]
Towball distance behind axle:	1.2 metres

Using these data, values of 3.702 metres and 18.92° are obtained for r_b and α respectively.

It will be noted from the graph that the rate of straightening falls with increasing alignment angle – very rapidly in the case of short trailers – and that all the curves intersect at an alignment angle equal to α.

It will also be observed that – for the part of each curve below the X axis – the rate of straightening is negative – meaning that the alignment angle is increasing rather than decreasing, resulting in a jack-knife situation. The point at which each curve crosses the X axis is, of course, the critical angle.

As has already been stated, a low positive rate of straightening makes manoeuvring very difficult – even if it doesn't result in a jack-knife. It is suggested that 10° per metre travelled should be regarded as a minimum full-lock straightening rate for acceptable manoeuvrability. In the example shown, this would limit the useable alignment angle to about 40° for a medium sized caravan, and to about 25° for a small camping trailer.

GRAPH 2 - Trailer Manoeuvrability (with Volvo V70)